

## THE SCALING EVOLUTION OF THE COSMOLOGICAL CONSTANT

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## ABSTRACT

In quantum field theory the parameters of the vacuum action are subject to renormalization group running. In particular, the “cosmological constant” is not a constant in a quantum field theory context, still less should be zero. In this paper we continue with previous work, and derive the particle contributions to the running of the cosmological and gravitational constants in the framework of the Standard Model in curved space-time. At higher energies the calculation is performed in a sharp cut off approximation. We assess, in two different frameworks, whether the scaling dependences of the cosmological and gravitational constants spoil primordial nucleosynthesis. Finally, the cosmological implications of the running of the cosmological constant are discussed.

## 1 Introduction

The cosmological constant (CC) problem [1] is, nowadays, one of the main points of attention of theoretical physics and astrophysics. The main reason for this is twofold: i) The recent measurements of the cosmological parameters [2] from high-redshift supernovae [3] and the precise data on the temperature anisotropies in the cosmic microwave background radiation (CMBR) [4, 5] offer unprecedentedly new experimental information on the model universe<sup>3</sup>; ii) A deeper understanding or even the final solution of the CC problem is one of the few things that theoretical physics can expect from the highly mathematized developments of the last decades: from strings and dualities to the semi-phenomenological Randall-Sundrum model and modifications thereof [7, 8, 9]. The very optimistic expectation includes also the prediction of the observable particle spectrum of the Standard Model. However, all attempts to deduce the small value of the cosmological constant

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<sup>3</sup>For a short review of the FLRW cosmological models with non-vanishing cosmological term, in the light of the recent observations, see e.g. [6].

from some sound theoretical idea, without fine tuning, failed so far and the anthropic considerations could eventually become a useful alternative to the formal solution from the first principles of field theory [1, 10].

In the present paper we continue earlier work on the scaling behavior of the CC presented in [11]. We look at the CC problem using the Renormalization Group (RG) and the well established formalism of quantum field theory in curved space-time (see, for example, [13, 12]). This way, certainly, does not provide the fundamental solution of the cosmological constant problem either. Nevertheless it helps in better understanding the problem and (maybe even more important) in drawing some physical consequences out of it. The CC problem arises in the Standard Model (SM) of the strong and electroweak interactions due to the spontaneous symmetry breaking (SSB) of the electroweak gauge symmetry and the presence of the non-perturbative QCD vacuum condensates. Both effects contribute to the vacuum energy density, and when the SM is coupled to classical gravity they go over to the so-called induced cosmological term,  $\Lambda_{ind}$ . The main induced contribution is the electroweak one, roughly given by  $\Lambda_{ind} \sim M_F^4$  – the fourth power of the Fermi scale  $M_F \equiv G_F^{-1/2}$ . Then the physical (observable) value of the CC (denoted by  $\Lambda_{ph}$ ) is the sum of the original vacuum cosmological term in Einstein equations,  $\Lambda_{vac}$ , and the total induced contribution,  $\Lambda_{ind}$ , both of which are individually unobservable. The CC problem manifests itself in the necessity of the unnaturally exact fine tuning of the original  $\Lambda_{vac}$  that has to cancel the induced counterpart within a precision (in the SM) of one part in  $10^{55}$ . These two: induced and vacuum CC's, satisfy independent renormalization group equations (RGE). Then, due to the quantum effects of the massive particles, the physical value of the CC evolves with the energy scale  $\mu$ :  $\Lambda_{ph} \rightarrow \Lambda_{ph}(\mu)$ . Remarkably, the running of the observable CC has an acceptable range, thanks to the cancellation of the leading contributions to the  $\beta$ -functions, which occurs automatically in the SM [11].

Here we are going to develop the same ideas further. The organization of the paper is as follows. In the next section, we review the renormalization of the vacuum action and show that there are no grounds to expect zero CC in this framework. In section 3, we clarify the source of the cancellation in the renormalization group equation for the induced CC in the SM. In section 4, we evaluate the value of the CC for higher energies, up to the electroweak (Fermi) scale and discuss the possible effect of the heavy degrees of freedom. After that, in section 5, it is verified whether the running of the CC spoils primordial nucleosynthesis in two possible frameworks. In section 6 we consider the scaling dependence of the Newton constant, and show that such dependence cannot be relevant even at the inflationary scales. In section 7 the role of the CC in the anomaly-induced inflation is discussed. Finally, in the last section we draw our conclusions.

## 2 Renormalization of the vacuum action

Since we are going to discuss the SM in relation to gravity, it is necessary to formulate the theory on the classical curved background. In order to construct a renormalizable gauge theory in an external gravitational field one has to start from the classical action which consists of three different parts <sup>4</sup>

$$S = S_m + S_{nonmin} + S_{vac} . \quad (1)$$

Here  $S_m$  is the matter action resulting from the corresponding action of the theory in flat space-time after replacing the partial derivatives by the covariant ones, Minkowski metric by the general

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<sup>4</sup>See e.g. [13, 12] for an introduction to renormalization in curved space-time.

metric and the integration volume element  $d^4x$  by  $d^4x\sqrt{-g}$ . For instance, the scalar kinetic term and the Higgs potential enter (1) through

$$S_{sc} = \int d^4x \sqrt{-g} \{ g^{\mu\nu} (D_\mu \Phi)^\dagger (D_\nu \Phi) - V_0(\Phi) \} , \quad (2)$$

where the derivative  $D_\mu$  is covariant with respect to general coordinate transformations and also with respect to the gauge transformations of the SM electroweak symmetry group  $SU(2)_L \times U(1)_Y$ . Thus

$$D_\mu = \nabla_\mu - ig T^i W_\mu^i - ig' Y B_\mu, \quad (3)$$

where  $\nabla_\mu$  is the coordinate-covariant part and the rest of the terms involve the standard gauge connections formed out of the electroweak bosons  $W_\mu^i$ ,  $B_\mu$  and the corresponding gauge couplings and generators. Other terms in the action involve similar generalization of the ordinary fermion, gauge and Yukawa coupling interactions of the SM. One of the novel features of the SM in a curved space-time background is the necessity of the “non-minimal term”

$$S_{nonmin} = \int d^4x \sqrt{-g} \xi \Phi^\dagger \Phi R, \quad (4)$$

involving the interaction of the  $SU(2)_L$  doublet of complex scalar fields  $\Phi$  with the curvature scalar  $R$ . Notice that for  $\xi = 0$  the gravitational field is still (minimally) coupled to matter through the metric tensor in the kinetic terms and in general to all terms of the Lagrangian density through the  $\sqrt{-g}$  insertion. With respect to the RG, one meets an effective running of  $\xi$  whose value depends on scale. This running has been studied for a variety of models (see [14, 12] and references therein).

Very important for our future considerations are the vacuum terms in the action, which are also required to insure renormalizability and hence repeat the form of the possible counterterms:

$$S_{vac} = \int d^4x \sqrt{-g} \left\{ a_1 R_{\mu\nu\alpha\beta}^2 + a_2 R_{\mu\nu}^2 + a_3 R^2 + a_4 \square R - \frac{1}{16\pi G_{vac}} R - \Lambda_{vac} \right\} \quad (5)$$

All the divergences in the theory (1) can be removed by the renormalization of the matter fields and couplings, masses, non-minimal parameter  $\xi$  and the bare parameters of the vacuum action  $a_{1,2,3,4}$ ,  $G_{vac}$  and  $\Lambda_{vac}$ .

Our main attention will be paid to the cosmological term in the vacuum action (5). Formally, the vacuum CC is required for renormalizability even in flat space-time. But then it is just a constant addition to the Lagrangian, which does not affect the equations of motion. In curved space-time, however, the situation is quite different, because the CC interacts with the metric through the  $\sqrt{-g}$  insertion. This was first noticed by Zeldovich [15], who also pointed out that there is a cosmological constant induced by matter fields.

Let us briefly describe the divergences in the vacuum sector. Suppose that one applies some covariant regularization depending on a massive parameter. For example, it can be regularization with higher derivatives [16, 17] with additional Pauli-Villars regularization in the one-loop sector. To fix ideas let us assume that all of the divergences depend on a unique regularization parameter  $\Omega$  with dimension of mass. Then, for the renormalizable theory (5), one faces three types of divergences in the vacuum sector:

i) Quartic divergences  $\sim \Omega^4$  for the cosmological term come from any field: massive or massless. As usual, these divergences must be subtracted by a counterterm. The renormalization

condition for the cosmological constant can be fixed at some scale (see below) in such a way that there should not be running due to these divergences. They fully cancel out against the counterterm; in other words, they can be “technically” disposed of once and forever. That is why the vacuum oscillations (zero-mode contributions) of the fields [1], although they could reach values as large as the Planck mass  $M_P$  to the fourth power, do not pose a severe problem and are usually considered as unimportant .

ii) Quadratic divergences are met in the Hilbert-Einstein and CC terms  $1/(16\pi G_{vac}) R + \Lambda_{vac}$  of Eq. (5). They can arise either from quadratically divergent graphs or appear as sub-leading divergences of quartic divergences. In the CC sector they are proportional to  $\Omega^2 m_i^2$ , where  $m_i$  are masses of the matter fields. Hence, massless fields do not contribute to these divergences. Quadratic divergences are removed in the same manner as the quartic ones. One can always construct the renormalization scheme in which no quadratic scale dependence remains after the divergences were canceled and the renormalization condition fixed.

iii) Finally, there are the logarithmic divergences. They show up in all sectors of Eq. (5), and come from all the fields: massless and massive. However, only massive fields contribute to the divergences of the CC and Hilbert-Einstein terms. This can be easily seen from dimensional analysis already. The logarithmic divergences are the most complicated ones, because even after being canceled by counterterms their effect is spread through the renormalization group.

What is the experimental situation at present? By virtue of the recent astronomical observations from high red-shift supernovae [3], the present-day density of matter and the CC are given by  $\rho_M^0 \simeq 0.3 \rho_c^0$  and  $\Lambda_{ph} \simeq 0.7 \rho_c^0$ , respectively, where

$$\rho_c^0 \equiv 3 H_0^2 / 8 \pi G_N \simeq 8.1 h_0^2 \times 10^{-47} \text{ GeV}^2 = \left( 3.0 \sqrt{h_0} \times 10^{-3} \text{ eV} \right)^4 \quad (6)$$

is the critical density [2]. Here the dimensionless number  $h_0 = 0.65 \pm 0.1$  [4, 5] defines the experimental range for the present day Hubble’s constant

$$H_0 \equiv 100 h_0 \text{ Km sec}^{-1} \text{ Mpc}^{-1} \simeq 2.13 h_0 \times 10^{-42} \text{ GeV} . \quad (7)$$

The next step is to choose the renormalization condition at some fixed energy scale. The choice of the scale  $\mu$  is especially relevant for the CC, because the latter is observed only at long, cosmic distances, equivalently at very low energies. We shall identify the meaning of  $\mu$  more properly later on, but for the moment it suffices to say that our considerations on the cosmological parameters always refer to some specific renormalization point, which we denote  $\mu_c$ . The choice of  $\mu_c$  must be such that at lower energies,  $\mu < \mu_c$ , there is no running. From the RG analysis we may expect [11]  $\Lambda_{ph} \sim \mu_c^4$  and so from the above experimental results we must have

$$\mu_c = \mathcal{O}(10^{-3}) \text{ eV} . \quad (8)$$

Since the renormalization of the CC is only due to the contributions of massive particles, one may guess that  $\mu_c$  must be of the order of the lightest particle with non-vanishing mass. In a minimal extension of the SM, it is the lightest neutrino (denoted here as  $\nu_1$ ), so we may expect  $\mu_c \approx m_{\nu_1} = \mathcal{O}(10^{-3}) \text{ eV}$ , which in fact holds good if we rely on the current results on neutrino masses [18]. Whether this is a coincidence or not cannot be decided at this stage, but one can offer RG arguments in favor of it [11]. Actually, from these results we collect, not one but three “cosmic coincidences”; i) The physical value of the CC is positive and of the order of  $\rho_M^0$ ; ii) The

density of matter is of the order of the critical density ( $\rho_M^0 \sim \rho_c^0$ ); and iii)  $m_{\nu_1}^4$  is of the order of  $\rho_M^0$  and so of order  $\Lambda_{ph}$ . Hence <sup>5</sup>

$$(\Lambda_{ph})^{1/4} \sim (\rho_c^0)^{1/4} = 3.0 \sqrt{\hbar_0} \times 10^{-3} \text{ eV} \sim m_{\nu_1} \quad (9)$$

As we shall see in the following, these coincidences are perhaps not independent, if the CC problem is addressed from the RG point of view.

At energies up to the Fermi scale,  $\mu \lesssim M_F$ , the higher derivative terms in (5) are not important for our considerations, and so the renormalized effective vacuum action at these low energies is just the Hilbert-Einstein action with a running cosmological and gravitational constants  $G_{vac}(\mu)$ ,  $\Lambda_{vac}(\mu)$ :

$$S_{HE}(\mu) = - \int d^4x \sqrt{-g} \left\{ \frac{1}{16\pi G_{vac}(\mu)} R + \Lambda_{vac}(\mu) \right\}. \quad (10)$$

In the last equation the dependence on  $\mu$  is governed by the renormalization group. At lower energies heavy particles decouple and do not, in principle, contribute to the running. However, this consideration must be handled with care in the CC framework. For a better understanding of the decoupling of heavy particles we have to remember that the decoupling mechanism [26] compares the mass of the particle inside the quantum loop with the energy of the particles in the external lines connected to this loop. Here, we are dealing with the vacuum diagrams and the gravitational field and therefore the relevant external legs to consider are the ones of gravitons<sup>6</sup>. The next problem is to evaluate the energy of these gravitons. Indeed, there is no general method to estimate the energy of the gravitational field, so we have some freedom at this point. One of the possibilities is the following. At low energy the dynamics of gravity is defined by the Einstein equations

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G_{ph} (T_{\mu\nu} + g_{\mu\nu} \Lambda_{ph}), \quad (11)$$

where  $G_{ph} = 1/M_P^2$  is the physical value of Newton's constant—see section 6. We may use the value of the curvature scalar ( $R$ ) as an order parameter for the gravitational energy, so the RG scale  $\mu$  can be associated with  $R^{1/2}$ . From eq.(11) we see that this is equivalent to take  $\mu \sim \sqrt{T_\mu^\mu/M_P^2}$ . But in the cosmological setting the basic dynamical equations refer to the scale factor  $a(t)$  of the Friedmann-Lemaître-Robertson-Walker (FLRW) metric [2], and so we must re-express the graviton energy in terms of it. The 00 component of (11) yields the well-known Friedmann-Lemaître equation

$$H^2 \equiv \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi}{3 M_P^2} (\rho + \Lambda_{ph}) - \frac{k}{a^2}. \quad (12)$$

The space curvature term can be safely set to zero ( $k = 0$ ), because the universe is very flat at present [2] and in general it may have undergone an inflationary period [19], so that  $k = 0$  effectively throughout the whole FLRW regime. The spatial components of (11), combined with the 00 component (12), yields the following dynamical equation for  $a(t)$ :

$$\ddot{a} = - \frac{4\pi}{3 M_P^2} (\rho + 3p - 2\Lambda_{ph}) a. \quad (13)$$

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<sup>5</sup>The contribution to  $\rho_M^0$  from light neutrinos is [2]  $\rho_M^{(\nu)} h_0^2 \simeq (m_{\nu_1}/92 \text{ eV}) \rho_c^0$ . In view of eq.(8),  $\rho_M^{(\nu)}$  is much smaller than  $m_{\nu_1}^4$ . Heavier neutrino species also contribute, but it is not clear why the full matter density happens to coincide (in order of magnitude) with the fourth power of the mass of the lightest species.

<sup>6</sup>The term graviton is used here in a generic sense referring to the presumed quantum of gravity as a field theory with a tensor potential, rather than to the gravitational waves.

In these equations  $\rho = \rho_M + \rho_R$  is the total energy density of matter and radiation, and  $p$  is the pressure. In the modern Universe  $p \simeq 0$  and  $\rho \simeq \rho_M^0$ . Moreover, from the recent supernovae data [3], we know that  $\Lambda_{ph}$  and  $\rho_M^0$  have the same order of magnitude as the critical density  $\rho_c^0$ . Therefore, the source term on the *r.h.s.* of (13) is characterized by a single dimensional parameter  $\sqrt{\rho_c^0/M_P^2}$ , which according to eq. (12) is nothing but the experimentally measurable Hubble's constant  $H_0$ . This is obviously consistent with the expected result  $\sqrt{T_\mu^\mu/M_P^2}$  in the general case because  $T_\mu^\mu \sim \rho_M^0 \sim \rho_c^0$  for the present-day universe. Therefore, we conclude that

$$\mu \sim R^{1/2} \sim H(t), \quad (14)$$

can be the proper identification for the RG scale  $\mu$  in the semiclassical treatment of the FLRW cosmological framework. Another possible choice (the one advocated in [11]) is to consider the critical density,

$$\mu \sim \rho_c^{1/4}(t), \quad (15)$$

as a typical energy of the Universe. During the hot stages of its evolution (see Section 5) this entails a direct association with temperature  $\mu \sim T$  [11, 21]. Although there might be other reasonable possibilities we will only consider these two “ $\mu$ -frames” in the present paper. In the following we will evaluate the running of the CC and Newton's constant with respect to the change in magnitude of the graviton energy and only after that we will discuss the scale at which these estimates might be applied and the differences between the two choices of  $\mu$ .

Let us postpone the discussion of the renormalization of the other terms in (5) until section 7 and concentrate now on the CC. As stated, the value of the CC is supposed to be essentially constant between  $\mu_c$  and the present cosmic scale  $H_0$ . Hence, one can impose the renormalization condition at  $\mu_c$ . As soon as one deals with the SM in curved space-time, the vacuum parameters including  $\Lambda_{vac}$ , should be included into the list of parameters of the SM. These parameters must be renormalized and their physical values should be implemented via the renormalization conditions. Exactly as for any other parameter of the SM, the values of  $\Lambda_{vac}$  and  $\xi, G_{vac}, a_{1,2,3,4}$  should result from the experiment. However, there is an essential difference between the CC and, say, masses of the particles. The key point of the CC problem is that there is another “induced” contribution  $\Lambda_{ind}$  to the CC along with the  $\Lambda_{vac}$ . The observable CC is the sum of the vacuum and induced terms

$$\Lambda_{ph} = \Lambda_{vac} + \Lambda_{ind}, \quad (16)$$

evaluated at the cosmic renormalization scale  $\mu_c$ .

The values of the electroweak parameters of the SM are defined from high energy experiments. The characteristic scale in this case is the Fermi scale  $M_F \equiv G_F^{-1/2} \simeq 293 \text{ GeV}$ . At the same time, due to the weakness of the gravitational force at small distances, there is no way to measure the vacuum parameters at this scale. However, recent astronomical observations are currently being interpreted as providing the right order of magnitude of the “physical” cosmological constant at present,  $\Lambda_{ph}$ , and it comes out to be non-zero at the 99% C.L. [3]. Now, the value of  $\Lambda_{ph}$  derived from these observations will be treated here as the value of the running parameter  $\Lambda_{ph}(\mu)$  evaluated at  $\mu = \mu_c$ .

Let us now review the mechanism for the induced CC, in the electroweak sector of the SM. In the ground (vacuum) state of the SM, the expectation value (VEV) of  $\Phi^\dagger \Phi$  in (2) will be denoted

$\langle \Phi^\dagger \Phi \rangle \equiv \frac{1}{2} \phi^2$ , where  $\phi$  is a classical scalar field. The corresponding classical potential reads

$$V_{cl} = -\frac{1}{2} m^2 \phi^2 + \frac{f}{8} \phi^4. \quad (17)$$

Shifting the original field  $\phi \rightarrow H^0 + v$  such that the physical scalar field  $H^0$  has zero VEV one obtains the physical mass of the Higgs boson:  $M_H = \sqrt{2} m$ . Minimization of the potential (17) yields the SSB relation:

$$\phi = \sqrt{\frac{2m^2}{f}} = v \quad \text{and} \quad f = \frac{M_H^2}{v^2}. \quad (18)$$

The VEV  $\langle \Phi \rangle \equiv v/\sqrt{2}$  gives masses to fermions and weak gauge bosons through

$$m_i = h_i \frac{v}{\sqrt{2}}, \quad M_W^2 = \frac{1}{4} g^2 v^2, \quad M_Z^2 = \frac{1}{4} (g^2 + g'^2) v^2, \quad (19)$$

where  $h_i$  are the corresponding Yukawa couplings, and  $g$  and  $g'$  are the  $SU(2)_L$  and  $U(1)_Y$  gauge couplings. The VEV can be written entirely in terms of the Fermi scale:  $v = 2^{-1/4} M_F \simeq 246 \text{ GeV}$ . From (18) one obtains the following value for the potential, at the tree-level, that goes over to the induced CC:

$$\Lambda_{ind} = \langle V_{cl} \rangle = -\frac{m^4}{2f}. \quad (20)$$

If we apply the current numerical bound  $M_H \gtrsim 115 \text{ GeV}$  from LEP II, then the corresponding value  $|\Lambda_{ind}| \simeq 1.0 \times 10^8 \text{ GeV}^4$  is 55 orders of magnitude greater than the observed upper bound for the CC – typically this bound is  $\Lambda_{ph} \lesssim 10^{-47} \text{ GeV}^4$ .

In order to keep the quantum field theory consistent with astronomical observations, one has to demand that the two parts should cancel with the accuracy dictated by the current data. This defines the sum (16). As shown by Eq.(20), the first term  $\Lambda_{ind}$  on the *r.h.s.* of (16) is not an independent parameter of the SM, since it is constructed from other parameters like the VEV of Higgs and couplings. On the contrary,  $\Lambda_{vac}$  is an independent parameter and requires an independent renormalization condition. From the quantum field theory point of view, the sequence of steps in defining the CC is the following: one has to calculate the value of  $\Lambda_{ind}$  at  $\mu_c$ , measure the value of the physical CC,  $\Lambda_{ph}$ , at the same scale, and choose the renormalization condition for  $\Lambda_{vac}$  in the form

$$\Lambda_{vac}(\mu_c) = \Lambda_{ph}(\mu_c) - \Lambda_{ind}(\mu_c). \quad (21)$$

The modern observations from the supernovae [3] tell us that the value of  $\Lambda_{ph}(\mu = \mu_c)$  is positive and has the magnitude of the order of  $\rho_c^0$ , that is about  $10^{-47} \text{ GeV}^4$ . This value should be inserted into the renormalization condition (21). From the formal point of view, everything is consistent. There is no reason to insist that the CC should be exactly zero, for it is measured to be nonzero by experiment. In principle, since the renormalization condition for the CC should be taken from the measurement, to insist on any other value, including zero, is senseless. But, the problem with the Eq. (16) is that the terms on the *r.h.s.* of it are 55 orders greater than their sum, so that one has to define  $\Lambda_{vac}(\mu_c)$  with the precision of 55 decimal orders. To explain this fantastic exactness is the CC problem. Of course, the fine tuning of 55 decimal numbers is difficult to understand, but zero CC would mean infinitely exact fine tuning, which would be infinitely hard to explain, at least

from the RG point of view. Let us compare the CC with any other parameter of the SM. Imagine, for instance, that we could isolate some particle like the electron or the top quark. Suppose also that we could measure its mass with the  $55^{th}$  order of magnitude precision. Then we meet a similar problem, because we would not be able to explain why the mass of this particle is exactly that one we measure. Indeed, for the electron (not to mention the top quark!) the  $55^{th}$  order of magnitude precision is not possible, so the exactness of the “fine tuning” for the CC really looks as something outstanding. However, this just manifests the fact that the cosmic scale where the measurement is performed, is quite different from the Fermi scale, where the second counterpart  $\Lambda_{ind}$  is defined. Therefore, the difference between the CC and the particle masses is that the first one can only be measured at the cosmic scale. Unfortunately, the problem of CC is deeper than that. Let us continue our comparison with the electron mass. It is known to be  $m_e = 0.51099906(15) MeV$ . But if it would be, say,  $m_e = 0.52 MeV$ , physics should be, perhaps, the same. At the same time, if we change, for example, the last 7 decimal points in the 55-digit number for the modern value of the CC, the energy density of the Universe would change a lot and the shape of the whole Universe would look very different. For instance, the Universe could be in a state of fast inflation. Thus, the problem of the fine tuning of the CC is much more severe than the prediction of particle masses.

The point is that we do not know why our Universe, with its small value of the CC, is what it is. This can be taken as a philosophic question, but if taken as a physical problem, it is really difficult to solve. At present, there are three main approaches. One of them supposes that there is some hidden symmetry which makes CC exactly zero, for instance, at the zero energy scale [7]<sup>7</sup>. Then one has to explain the change to the nonzero value of the CC at the present cosmic scale. The second possibility is the anthropic hypothesis, which supposes that our Universe is such as it is because we are able to live in it and study it. An extended version supposes multiple universes and challenges one to calculate the probability to meet an appropriate Universe, available for doing theoretical physics [23]. The third approach is called quintessence and assumes that the CC is nothing but a scalar field providing negative pressure and a time-varying, spatially fluctuating energy density [24].

Let us consider the scale dependence in the RG framework. When the Universe evolves, the energy scale changes and it is accompanied by the scale dependence (running) of the physical quantities like charges and masses. Taking the quantum effects into account, one cannot fix the CC to be much smaller than  $10^{-47} GeV^4$ , because such a constraint would be broken by the RG at the energies comparable to the small neutrino masses [11]. In fact, we can accept (21) as an experimental fact, without looking for its fundamental reasons. Then, the following questions appear: i) Is the running of the CC consistent with the standard cosmological model?; ii) Which kind of lessons can we learn from this running?

As we have shown in [11] the running for the observable CC really takes place. The RGE for the parameter  $\Lambda_{vac}$  is independent from the RG behavior of the induced value  $\Lambda_{ind}$ , and as a result the sum (16) diverges from its value at the fixed cosmic scale (21). It is important to notice that the running of the physical (observable) CC signifies that one cannot have zero CC during the whole life of the Universe because a CC of the order of the  $\beta$ -function would appear at the neutrino

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<sup>7</sup>One can, also, mention the interesting recent papers [22] where the methods of condensed matter physics were called to solve the CC problem. According to [22], the  $CC=0$  in the infinitely remote future (for the open Universe) follows from the condition of equilibrium for the matter filling this Universe. While being a useful observation, this does not solve the fundamental CC problem. Indeed, the presence of the CC causes the matter filling the Universe to be in a non-equilibrium state in the remote future. Thus, to postulate as a fundamental principle that the matter *should* eventually reach the state of equilibrium is just a version of the standard fine tuning of the CC.



scale. For this reason, popular quintessence models, which are called to mimic the CC cannot "explain" the observed value of the CC [10]. With or without quintessence, one has to choose the renormalization condition for the vacuum CC. The only difference is that, in (21), one has to add the quintessence contribution to  $\Lambda_{ind}$ , and the fine tuning becomes a bit more complicated. Hence, quintessence can be useful to solve some other problem, but in our opinion it does not simplify at all the CC problem.

### 3 Running of vacuum and induced counterparts

The RGE for the effective action can be formulated in curved space-time in the usual form [14, 12]:

$$\left\{ \mu \frac{\partial}{\partial \mu} + (\beta_p - d_p p) \frac{\partial}{\partial p} + \gamma_i \int d^4 x \sqrt{-g} \Phi_i \frac{\delta}{\delta \Phi_i} \right\} \Gamma[g_{\alpha\beta}, p, \Psi_i, \mu] = 0. \quad (22)$$

Here the  $\beta$ -functions for all the couplings, vacuum parameters and masses of the theory (generically denoted by  $p$ ) and the  $\gamma_i$ -functions of all the matter fields  $\Phi_i$  are defined in the usual way.

Equation (22) enables one to investigate the running of the coupling constants and also the behavior of the effective action in a strong gravitational field, strong scalar field and other limits [12]. We are interested in the running (general dependence on  $\mu$ ) of the CC and Newton's constant. Also, we shall consider the RGE's for other parameters when necessary.

To study the running of the physical CC and also, in a subsequent section, for the Newton constant, we need the  $\beta$ -functions for the scalar coupling constant  $f$ , for the Higgs mass parameter  $m$  and for the dimensional parameters  $G_{vac}$ ,  $\Lambda_{vac}$  of the vacuum action. At this stage we write down the full RGE's without restrictions on the contributions of heavy particles. These restrictions will be imposed later, when we evaluate the running at different energies. Taking into account all the fields entering the SM we arrive at the following RGE:

$$\begin{aligned} (4\pi)^2 \frac{dm^2}{dt} &= m^2 \left( 6f - \frac{9}{2} g^2 - \frac{3}{2} g'^2 + 2 \sum_{i=q,l} N_i h_i^2 \right), & m(0) &= m(\mu = M_F) \equiv m_F, \\ (4\pi)^2 \frac{df}{dt} &= 12f^2 - 9fg^2 - 3fg'^2 + \frac{9}{4} g^4 + \frac{3}{2} g^2 g'^2 + \frac{3}{4} g'^4 \\ &\quad + 4 \sum_{i=q,l} N_i h_i^2 (f - h_i^2) & f(0) &= f(\mu = M_F) \equiv f_F. \end{aligned} \quad (23)$$

Here  $h_i = h_{l,q}$  are the Yukawa couplings for the fermions: quark  $q = (u, \dots, t)$  and lepton  $l = (\nu_e, \nu_\tau, \nu_\mu, e, \mu, \tau)$  constituents of the SM. Furthermore,  $t = \ln(\mu/M_F)$ , and  $N_i = 1, 3$  for leptons and quarks respectively. The boundary conditions for the renormalization group flow are imposed at the Fermi scale  $M_F$  for all the parameters, with the important exception of  $\Lambda_{vac}$ . Then the  $SU(2)_L$  and  $U(1)_Y$  gauge couplings at  $\mu = M_F$  are  $g_F^2 \approx 0.4$  and  $g_F'^2 = g_F^2 \tan^2 \theta_W \simeq 0.12$ . Here  $\theta_W$  is the weak mixing angle, and at the Fermi scale  $\sin^2 \theta_W \simeq 0.23$ .

Taking the renormalization conditions into account, the solution of (23) for  $m$  can be written in the form:

$$m^2(t) = m_F^2 U(t), \quad (24)$$

with

$$U(t) = \exp \left\{ \int_0^t \frac{dt}{(4\pi)^2} \left[ 6f(t) - \frac{9}{2} g^2(t) - \frac{3}{2} g'^2(t) + \sum_i N_i h_i^2(t) \right] \right\}, \quad (25)$$

where the couplings satisfy their own (well known) RGE [25]. The one-loop  $\beta$ -function for the vacuum CC gains contributions from all massive fields, and can be computed in a straightforward way by explicit evaluation of the vacuum loops (Cf. *Fig.1*). In particular, the contribution from the complex Higgs doublet  $\Phi$  and the fermions is (for  $\mu \gtrsim M_F$ )

$$(4\pi)^2 \frac{d\Lambda_{vac}}{dt} = \beta_\Lambda \equiv 2m^4 - 2 \sum_i N_i m_i^4, \quad \Lambda_{vac}(0) = \Lambda_0, \quad (26)$$

where the sum is taken over all the fermions with masses  $m_i$ . In the last formula we have changed the dimensionless scaling variable into  $t = \ln(\mu/\mu_c)$  because, as we have already argued above, the renormalization point for  $\Lambda_{vac}$  is  $\mu = \mu_c$ . Taking (24) into account, the solution for the vacuum CC is

$$\Lambda_{vac}(t) = \Lambda_0 + \frac{2m_F^4}{(4\pi)^2} \int_0^t U^2(t) dt - \frac{2}{(4\pi)^2} \sum_i N_i \int_0^t m_i^4(t) dt, \quad (27)$$

where the running of  $m_i(t)$  is coupled to that of  $m^2(t)$  and the corresponding Yukawa couplings. The scale behavior of  $\Lambda_{ind}$  depends on the running of  $m(t)$  and  $f(t)$ , so that from Eq.(20) we have

$$\Lambda_{ind}(t) = -\frac{m_F^4 U^2(t)}{2f(t)}, \quad (28)$$

where  $f(t)$  is solution of Eq. (23). Although the value of the Higgs mass is not well under control at present, and therefore the initial data for  $f$  is unknown, this uncertainty does not pose a problem for the running of the CC, especially at low energies where the heavy degrees of freedom play an inessential role. Eqs. (27), (28) enable one to write the general formula for the scale dependence of the CC, in a one-loop approximation:

$$\Lambda_{ph}(t) = \Lambda_0 - \frac{m_F^4 U^2(t)}{2f(t)} + \frac{2m_F^4}{(4\pi)^2} \int_0^t U^2(t) dt - \frac{2}{(4\pi)^2} \sum_i N_i \int_0^t m_i^4(t) dt, \quad (29)$$

where  $t = \ln(\mu/\mu_c)$ . An important point concerning the RGE is the energy scale where they actually apply. This is especially important in dealing with the CC problem, since this problem is seen at the energies far below the Standard Model scale ( $\mu_c \ll M_F$ ). The corresponding  $\beta$ -functions  $\beta_{\Lambda_{vac}}, \beta_m, \beta_{h_i}, \beta_f \dots$  depend on the number of active degrees of freedom. These are the number of fields whose associated particles have a mass below the typical energy scale  $\mu$  of the gravitons (e.g., external legs of the diagrams which must be, indeed, added at *Fig. 1*), because at sufficiently small energies one can invoke the decoupling of the heavier degrees of freedom [26]. Equation (29) is normalized according to (21) such that the quantity  $\Lambda_{ph}(0)$  exactly reproduces the value of the CC from supernovae data. Therefore, it should be clear that in our framework the relevant CC at present is not the value of (28) at  $\mu = M_F$  but, instead, that of (29) at  $\mu = \mu_c \ll M_F$ . The value of the CC at the Fermi scale will be computed below within our approach.

First we will be interested in the scaling behavior of  $\Lambda_{ph}$  starting from the low energy scale  $\mu \sim \mu_c$ . One may expect that the lightest degrees of freedom of the SM, namely the neutrinos, are the only ones involved to determine the running  $\Lambda_{ph}$  at nearby points  $\mu \gtrsim \mu_c$ . Thus, let us suppose that all other constituents of the SM decouple, including the heavier neutrino species (see below) and of course all other fermions, scalar and gauge bosons. For example, the electron (which is the next-to-lightest matter particle after the neutrino) has a mass which is  $10^8$  times heavier than the assumed mass for the lightest neutrino species [18]. Within this Ansatz, we have to take

into account only light neutrino loops. Moreover, we can safely neglect the running of the mass  $m(t)$  and coupling  $f(t)$ , and attribute their values at the Fermi scale to them. For, the effect of their running at one loop is of the same order as the second loop corrections to the running of  $\Lambda_{vac}$  and  $\Lambda_{ind}$ , because they are proportional to the same neutrino Yukawa couplings.

Substituting (23) into the expression (20), we arrive at the following equation:

$$\frac{d\Lambda_{ind}}{dt} = \frac{m^4}{2f^2} \frac{df}{dt} - \frac{m^2}{f} \frac{dm^2}{dt} = -\frac{1}{(4\pi)^2} \cdot \frac{2m^4}{f^2} \cdot \sum_j h_j^4 = -\frac{2}{(4\pi)^2} \sum_j m_j^4. \quad (30)$$

Here we have used the fact that in the SM the coefficient  $m^4 h_j^4 / f^2$  is nothing but the fourth power of the fermion mass,  $m_j^4$  - as it follows from Eqs. (18) and (19). In fact, the *r.h.s.* of (30) looks like a miracle occurring in the SM, because it exhibits a cancellation of the leading  $m^4 h_j^2 / f$  terms. These terms are 28 orders of magnitude greater than the remaining ones  $m^4 h_j^4 / f^2$  in the case of the lightest neutrinos (as seen by using Eqs. (18,19), the ratio of the two is  $f/h^2 = M_H^2/2m_\nu^2 \sim 10^{28}$  for  $m_\nu \sim 10^{-3} \text{ eV}$ ). Without this cancellation the range of the running would be unacceptable, and the fine tuning of the CC incompatible with the standard cosmological scenario (see section 5). This does not happen in the SM due to the mentioned cancellation, the origin of which will be explained at the end of this section.

Taking only neutrino contributions into account, we see from (26) and (30) that the RGE for the vacuum and induced CC are identical<sup>8</sup>. Hence the running of the physical CC is governed by the equation

$$(4\pi)^2 \frac{d\Lambda_{ph}}{dt} = -4 \sum_j m_j^4. \quad (31)$$

Here, as before, we have normalized  $t$  such that  $t = \ln(\mu/\mu_c)$ . Now, let us make some comment on the cancellation in the one-loop contribution (30).

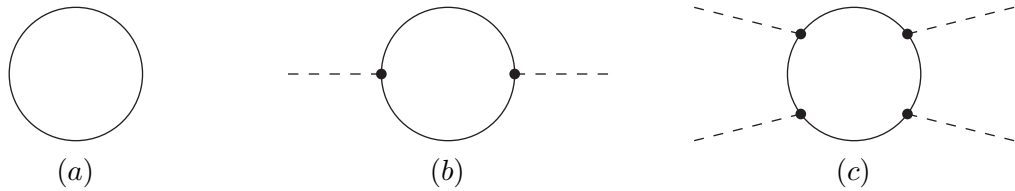


Figure 1. Three relevant diagrams: (a) The one-loop contributions to the vacuum part are just bubbles without external lines of matter; (b) The one-loop two-point function contributing to the induced part of the CC. (c) The one-loop four-point function contributing to the induced part.

When investigating the running around  $\mu \sim \mu_c$ , one has to omit all the diagrams with the closed loops of heavy particles. Then, for the running of the vacuum CC, one meets only closed

<sup>8</sup>Of course, this depends on the approximation of constant mass  $m$  and coupling  $f$ , which we use. At low energies the running of  $m$  and  $f$  are negligible.

neutrino loops without external tails. In the induced sector, however, there are two sorts of neutrino diagrams (see *Fig.1*): (b) the ones contributing to the renormalization of the Higgs mass, and (c) the ones contributing to the  $\phi^4$ -vertex. In the general case and in dimensional regularization one has

$$\begin{aligned}\phi_0 &= \mu^{(n-4)/2} Z_1^{1/2} \phi = \mu^{(n-4)/2} (1 + \frac{1}{2} \delta Z_1) \phi, \\ m_0^2 &= Z_2 m^2 + \delta Z_3 m_\nu^2 = (1 + \delta Z_2) m^2 + \delta Z_3 m_\nu^2, \\ f_0 &= \mu^{4-n} Z_f f = \mu^{4-n} (1 + \delta Z_f) f.\end{aligned}\tag{32}$$

where  $m_\nu$  is the neutrino mass,  $\delta Z_1$ ,  $\delta Z_2$  and  $\delta Z_3$  are divergent one-loop contributions coming from the diagrams in *Fig.1b*, and  $\delta Z_f$  comes from the diagram in *Fig. 1c*. At the low-energy cosmic scale  $\mu_c$  the heavy fields do not contribute, so that  $\delta Z_2 = 0$ . But of course at the Fermi scale a non-trivial  $\delta Z_2$  contribution must be properly taken into account. Furthermore, at this scale there is an extended list of one-loop diagrams – involving the effects from all fermions, Higgs and gauge bosons of the SM– from which the general RGE (23) were derived. However, as we said above, when calculating the  $\beta$ -function for  $f$  at low energy, we may restrict ourself to the diagrams in *Fig.1*. As a result we have the  $h_\nu^4$ -order contribution to the  $\phi^4$ -vertex from the diagram in *Fig. 1c* plus a  $f h_\nu^2$ -order contribution from the tree-level  $\phi^4$ -vertex including the mass counterterm insertion  $\delta Z_3$  on any of the external legs. This is the way the “big” terms proportional to  $f h_\nu^2$  enter the calculation. As a consequence, when computing the  $\beta$ -function for  $\Lambda_{ind}$  in Eq.(30) the  $h_\nu^2$ -order contribution to  $m^2$  from *Fig. 1b* cancels against the vertex diagram containing the  $\delta Z_3$  insertion. Since both terms have the same origin, it is not a real miracle that they cancel out in the RGE for  $\Lambda_{ind}$ . The upshot is that only the  $h_\nu^4$ -order contribution from *Fig. 1c* remains. Since  $h_\nu^4$  is very small, the running of  $\Lambda_{ph}$  has an acceptable range. As for the higher loop diagrams, it is easy to check that these diagrams come with extra factors of  $h_\nu^2$ , and this renders them much smaller than the one-loop contributions.

## 4 The running of CC at higher energies

In the previous section we have discussed the scaling evolution of the CC at low energies in the region just above  $\mu_c$  assuming that only the lightest massive degrees of freedom are active. The study of the heavy degrees of freedom at higher energies meets several difficulties. Two main problems are the following: 1) one is the contribution of heavy particles at the energies near their mass, 2) the other is the “residual” effects from the heavy particles at energies well below their mass. The quantum effects of the massive particles are, in principle, suppressed at low energies [26], so that in the region below the mass of the particle its quantum effects become smaller, besides, they are not related to the UV divergences. At this point we need the relation between the IR and the UV regions. The best procedure to solve 1) would be to extend the Wilson RG for the quantitative description of the threshold effects, and to solve 2) we would need a mass-dependent RG formalism. But, since both of these formalisms are too cumbersome for an investigation at this stage, we will first of all tackle the problem by applying the standard “sharp cut-off” approximation within the minimal subtraction (MS) scheme, namely the contribution of a particle will be taken into account only at the energies greater than the mass of this particle<sup>9</sup>. Subsequently, we will roughly estimate

<sup>9</sup>It is well-known that the decoupling of heavy particles does not hold in a mass-independent scheme like the MS, and for this reason they must be decoupled by hand using the sharp cut-off procedure [29].

the potential modifications of this approach induced by the heavy particles.

We start by evaluating the successive contributions to the CC up to the Fermi scale  $M_F$  within the sharp cut-off approximation. The calculations are performed similarly to the neutrino case at low energy. The result is that the  $\beta_\Lambda$ -function in Eq.(26) gets, in the presence of arbitrary degrees of freedom of spin  $J$  and non-vanishing mass  $M_J$ , a corresponding contribution of the form

$$\beta_\Lambda = (-1)^{2J} (J + 1/2) n_c n_J M_J^4, \quad (33)$$

with  $(n_c, n_{1/2}) = (3, 2)$  for quarks,  $(1, 2)$  for leptons and  $(n_c, n_{0,1}) = (1, 1)$  for scalar and vector fields. The particular case of the Higgs contribution in Eq.(26) is recovered after including an extra factor of 4 from the fact that there are four real scalar fields in the Higgs doublet of the SM. Notice that this result is consistent with the expected form  $(1/2) M_H^4$  as the physical mass of the Higgs particle is  $M_H = \sqrt{2} m$ . The values of the CC at different scales, within our approximation, can be easily computed using the current SM inputs [27]. In particular we take  $M_H = 115 \text{ GeV}$  and  $m_t = 175 \text{ GeV}$ . The numerical results are displayed in Table 1. Notice that the last row gives the CC at the Fermi scale  $M_F$ . This value follows from integrating the RGE with the assumption that the masses have their values at the Fermi scale. From the formulae in Sec. 3 we obtain, after a straightforward calculation,

$$(4\pi)^2 \frac{d\Lambda_{ph}}{dt} = \frac{1}{2} M_H^4 + 3M_W^4 + \frac{3}{2} M_Z^4 - 12m_t^4 + \frac{3M_F^4}{32} \left( 1 + \frac{1}{2 \cos^4 \theta_W} \right) g^4 \quad (\mu \gtrsim M_F) \quad (34)$$

We point out that in all cases the contribution from the vacuum and induced parts is the same to within few percent at most.

d.o.f.	$m \text{ (GeV)}$	$\Lambda_{ph} \text{ (GeV}^4\text{)}$	$ \Lambda_{ph} /m^4$
$\nu_{\tau,\mu}$	$\approx 10^{-9}$	$\approx 10^{-47}$	$\mathcal{O}(10^{-11})$
e	$5 \times 10^{-4}$	$\approx -10^{-37}$	$\mathcal{O}(10^{-24})$
u	$5 \times 10^{-3}$	$-3.6 \times 10^{-15}$	$5.8 \times 10^{-6}$
d	0.01	$-3.3 \times 10^{-11}$	$3.3 \times 10^{-3}$
$\mu$	0.105	$-1.8 \times 10^{-9}$	$1.5 \times 10^{-5}$
s	0.3	$-3.2 \times 10^{-6}$	$4 \times 10^{-4}$
c	1.5	$-9.9 \times 10^{-4}$	$2 \times 10^{-4}$
$\tau$	1.78	-0.065	$6.7 \times 10^{-3}$
b	5	-0.33	$5.3 \times 10^{-4}$
$W$	80	-132	$3.2 \times 10^{-6}$
$M_F$	293	$-8.8 \times 10^{+7}$	0.012

Table 1. The numerical variation of  $\Lambda_{ph}$  at different scales  $\mu$ . Each scale is characterized by the mass  $m$  of the heaviest, but active, degrees of freedom (d.o.f.). In the last column, the value of  $\Lambda_{ph}(\mu)$  is presented also measured in the units of the fourth power of the natural mass scale  $\mu = m$ . Due to the lack of knowledge of the various neutrino masses, we have given only the order of magnitude of  $\Lambda_{ph}$  for the second row.

We suppose that the heavy couple of neutrinos ( $\nu_\mu, \nu_\tau$ ) have masses three orders greater than the masses of the electron and sterile neutrino (if available). These light neutrinos are assumed with

masses of  $\mathcal{O}(10^{-3})\text{ eV}$ , namely of order of the square root of the typical mass squared differences obtained in the various neutrino experiments [18]. Since the available data about the neutrino masses is not exact, their contribution is indicated only as an order of magnitude. In fact, all of the numbers in this Table are estimates, because of the reasons mentioned above. Let us make some remarks concerning the values of  $\Lambda_{ph}$  at different scales.

*First.* The breaking of the fine tuning between induced and vacuum CC's becomes stronger at higher energies. Even so, it is well under control because it is highly tamed by the automatic cancellation mechanism in (30). Remarkably,  $\Lambda_{ph}$  becomes negative from the heavy neutrino scale upwards, while its absolute value increases dramatically and achieves its maximum at the end point of the interval, the Fermi scale. Notice that at this scale we recover a physical value for the CC around  $10^8 \text{ GeV}^4$ , which is of the order of the one obtained from the naive calculation based on only the (tree-level) induced part, Eq.(20). However, in our framework the value at  $\mu = M_F$  is consistently derived from a physical CC of order  $10^{-47} \text{ GeV}^4$  at the cosmic scale  $\mu_c$ .

*Second,* the dimensionless ratio  $\Lambda_{ph}/m^4$  suffers from “jumps” at the different points. Such “jumps” occur at the particle thresholds when the new, heavier, degrees of freedom start to contribute to the running. Obviously, this is an effect of the sharp cut-off approximation. In a more precise scheme one has to switch on the contributions of the heavy particles in a smoother way (e.g. with the aid of a fully-fledged mass-dependent scheme), and then the scale dependence of the observable CC would be also smooth. Another possible drawback, although certainly not inherent to our approach as it is of very general nature, is the fact that our estimate for the CC at the scale of the light quark masses may be obscured by non-perturbative effects which are difficult to handle. Our rough approximation, however, should suffice to conclude that the relative cosmological constant  $\Lambda_{ph}/m^4$ , at energies above the heavy neutrino masses up to the Fermi scale, has a magnitude between  $10^{-2}$  and  $10^{-6}$ .

*Third.* One can suppose that there is some (yet unknown) fundamental principle, according to which the CC is exactly zero at the infinitely small energy (far IR), that corresponds to the thermodynamical equilibrium by the end of the evolution of the Universe. It is interesting to verify whether the change from the non-zero value of CC at present to zero CC at far IR could be the result of the RG running similar to that we have discussed above. It is not forbidden at all the existence of the light scalar with the mass similar to lightest neutrino mass of slightly heavier than the neutrino mass. In this case the running of the CC could be different from the one presented in the Table 1, in particular the first line or lines of this Table might change the sign. Only experiments can tell us whether this possibility is real or not. But, such a scalar can not change the value of CC at far IR, because this scalar has to decouple at the energies comparable  $\mu_c$  (simultaneously to neutrino) and can not affect physics at the scales below  $H_0$ . The only one possibility to produce the running of CC below  $H_0$  is to suppose the existence of some super-light scalar with the mass  $m_{sls} \ll H_0$ . Now, since the observed value of the CC  $\Lambda \approx \mu_c^4$  is much greater than  $m_{sls}^4$ , the only possibility is to have a huge number of copies for such scalars. On the other hand, there are some serious restrictions for the number of the super-light scalars, as we shall see in section 7. Hence, the possibility of having zero CC at far IR and non-zero now, due to the RG, does not look realistic.

*Fourth.* In the previous considerations, the contributions from the heavy particles of mass  $M$  at energies  $\mu \ll M$  are in principle suppressed by virtue of the decoupling theorem [26]<sup>10</sup>. However,

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<sup>10</sup>It is well-known that the decoupling of heavy particles does not hold in a mass-independent scheme like the MS, and for this reason they must be decoupled by hand using the sharp cut-off procedure above described [29].

one may take a critical point of view and deem for a moment that their effect could perhaps be not fully negligible in the context of the CC problem. In this case the modifications of the previous picture will also depend on the possible choices for the RG scale  $\mu$  which, as we have seen in Section 2, is not a completely obvious matter in the cosmological scenario. Whether these heavy mass terms are eventually relevant or not is not known, but one can at least discuss this possibility on generic grounds. Under this hypothesis the heavy particle effects emerging in a mass-dependent RG scheme would lead to a new RG equation for  $\Lambda_{ph}$  in which the *r.h.s.* of eq. (34) ought to be modified by additional dimension-4 terms involving the scale  $\mu$  itself, namely terms like  $\mu^2 M^2$ . This can be guessed from the fact that in a mass-dependent subtraction scheme [29] a heavy mass  $M$  enters the  $\beta$ -functions through the dimensionless combination  $\mu/M$ , so that the CC being a dimension-4 quantity is expected to have a  $\beta$ -function corrected as follows:

$$\beta(m, \frac{\mu}{M}) = a m^4 + b \left(\frac{\mu}{M}\right)^2 M^4 + \dots \quad (35)$$

where  $a, b$  are some coefficients and the dots stand for terms suppressed by higher order powers of  $\mu/M \ll 1$ . Therefore, if we now take e.g. eq. (14) as a physical definition of  $\mu$  in the gravitational context, eq.(35) would lead to a modified RG equation of the generic form

$$(4\pi)^2 \frac{d\Lambda_{ph}}{dt} = \sum_i a_i m_i^4 + \sum_j b_j H^2 M_j^2 + \dots \quad (36)$$

where  $m_i$  and  $M_j$  are the masses of the light and heavy degrees of freedom respectively, and the coefficients  $b_i$  will depend on the explicit mass-dependent computation, but can be expected to be of the order of the original coefficients  $a_i$  in the mass-independent scheme. The heaviest masses  $M_i$  in the SM framework are of order  $M_F$  and correspond to the Higgs and electroweak gauge bosons, and the top quark. In this setting the maximum effect from the heavy mass terms is of order  $\sim H^2 M_F^2$ . Nevertheless if we evaluate the *r.h.s.* of eq.(36) for the present epoch of our universe, there is no significant contribution other than the residual one because  $H_0 \ll m_i$  for any SM particle. So from eq.(7) we infer that the contribution to  $\Lambda_{ph}$  from these terms in the modern epoch is of order  $10^{-82} GeV^4$  at most, i.e. completely negligible. From this fact two relevant conclusions immediately ensue: first, that at present the CC essentially does not run (a welcome fact if we wish to think of  $\Lambda_{ph}$  as a “constant”), and second that its value  $\sim 10^{-47} GeV^4$  was fixed at a much earlier epoch when there was perhaps some active degree of freedom, like the lightest neutrino (8), and/or when the heavy mass contributions themselves were of order  $10^{-47} GeV^4$ . In contrast to these nice results, if we now take the alternative  $\mu$  frame (15) we find that the contributions  $\mu^2 M_i^2$  from the heavy particles could be much more important even at the modern epoch, rendering a yield of order  $m_{\nu_1}^2 M_F^2 / (4\pi)^2 \sim 10^{-21} GeV^4$  that could distort in a significant way the analysis made in the sharp cut-off approximation – unless one arranges for an additional fine tuning among the various  $\mu^2 M_i^2$  contributions [21]. As we shall see in the next section, the choice of  $\mu$  can be relevant not only for the contemporary epoch, but also for the implications in the early stages of our Universe.

## 5 Implications for the nucleosynthesis

The first test for the reliability of our effective approach comes from the primordial nucleosynthesis calculations. The standard version of these calculations implies that the total energy density

$\rho = \rho_R + \rho_M$  from radiation and matter fields is dominating over the density of vacuum energy:  $\rho \gg \Lambda_{ph}$ . In practice, it suffices to verify that  $\rho_R \gg \Lambda_{ph}$  because the radiation density is dominant at the nucleosynthesis epoch. So, we have to check what is the relation between the CC and the energy density  $\rho_R$  at the temperature around  $T_n = 0.1 \text{ MeV}$ , which is the most important one for the nucleosynthesis [2]. If we compare this energy with the electron mass  $m_e \simeq 0.5 \text{ MeV}$  and look at the Table 1 above, the plausible conclusion is that the CC is very small and cannot affect the standard nucleosynthesis results.

However, one has to remind that the nucleosynthesis already starts at the temperature  $10^{10} \text{ K} \simeq 1 \text{ MeV} \simeq 2m_e$ . At earlier stages the entropy is so high that the relevant reactions are suppressed by the high value of the photon-to-baryon ratio [2]. According to our previous analysis, that scale of energies is characterized by a fast growth of the negative CC due to the electron vacuum effects, and above  $(5 - 10) \text{ MeV}$  there is an even greater enhancement due to the light quark contributions.

Let us now evaluate the energy density of radiation of the Universe at these temperatures,  $\rho_R(T)$ . It can be obtained from the energy density of a black body at a temperature  $T$ . In units where Boltzman's constant is one, it reads

$$\rho_R(T) = \frac{\pi^2}{30} g_* T^4, \quad (37)$$

where  $g_* = 2$  for photons, and  $g_* = 3.36$  after including the three neutrino species – if considered massless or at least with a mass much smaller than  $T$ . Incidentally, for the density of microwave background photons at the present relic temperature  $T_0 \simeq 2.75 \text{ K} = 2.37 \times 10^{-4} \text{ eV}$  it gives  $\rho_{\text{CMBR}} = 2.5 \times 10^{-5} h_0^{-2} \rho_c^0$ , i.e. at most one ten-thousandth of the critical density today ( $h_0 > 0.5$ ) –see eq.(8). However, at very high energies the density of radiation was dominant. Thus, at the typical energy of the nucleosynthesis,  $T_n = 10^{-4} \text{ GeV}$ , we get

$$\rho_R(T_n) \simeq 1.1 \cdot 10^{-16} \text{ GeV}^4. \quad (38)$$

The relevant issue at stake now is to check whether the CC at this crucial epoch of the history of our universe was smaller, larger or of the same order of magnitude as the radiation energy density. If larger, it could perturb the nucleosynthesis and of course this could not be tolerated. Again the analysis may depend on the particular  $\mu$  choices (14) or (15) for the RG scale  $\mu$ , as well as on the inclusion or not of the heavy mass terms from eq.(35). Let us forget for the moment about these terms and start with the definition (15). Since we now find ourselves in the radiation dominated epoch, the typical energy density will be defined by the temperature  $T$ . So in this approach we may set  $\mu \sim T$  and compare (38) with the value of  $\Lambda_{ph}(\mu)$  obtained from its scaling evolution with  $\mu \sim T$ . This can be done by looking at Table 1. Notice that  $T$  is of order of  $m$  when a particle species of mass  $m$  is active, and so the last column of Table 1 basically gives the ratio  $|\Lambda_{ph}(\mu = M)|/\rho_R(T = M)$  up to a factor  $g_*\pi^2/15$  of order one. From the Table we realize that the result (38) is about 21 orders of magnitude bigger than the CC generated by the “heavy” neutrino effects up to the scale  $\mu \sim m_e$ . Thus, in the framework of our sharp cut-off approximation, the running of the CC cannot affect the nucleosynthesis. However, the situation is not that simple, because  $T_n$  is very close to the electron mass, and the contribution to the CC from this “heavy” particle may become important at the earlier stages of the nucleosynthesis. In order to see this, let us derive the density  $\rho_R$  for the upper energy end of the nucleosynthesis interval. Using (37) we arrive at the estimate  $\rho_R(T = m_e = 5T_n) \approx 7 \times 10^{-14} \text{ GeV}^4$  whereas the CC at  $\mu = m_e$  is of order  $10^{-37} \text{ GeV}^4$ . For even higher energies there is a dramatic enhancement of the CC at  $\mu \gtrsim m_u$  where  $\Lambda_{ph}$  becomes



of order  $10^{-15} GeV^4$  whereas  $\rho_R(T = m_u = 50T_n) \approx 7 \times 10^{-10} GeV^4$ . Still, in this case the CC is five-six orders of magnitude smaller than  $\rho_R$ . So in all cases  $\Lambda_{ph} \ll \rho_R$ , and this result should not depend on the sharp cut-off approximation. However, the situation changes dramatically if the heavy mass terms  $\mu^2 M_i^2$  would be present at all in the RGE. Their presence could be in trouble with nucleosynthesis due to induced contributions of order  $T_n^2 M_F^2 / (4\pi)^2 \sim 10^{-6} GeV^4$  which are much larger than (38). Quite in contrast, the choice (14) seems to be completely safe, both with and without heavy mass terms. In fact, from eqs.(37) and (12) Hubble's constant at the nucleosynthesis time is found to be  $H_n \sim 10^{-27} GeV$  and so  $H_n^2 M_F^2 / (4\pi)^2 \sim 10^{-51} GeV^4$ , which is much smaller than (38) and than the present day value of the CC. In short, in the absence of the heavy mass terms  $\mu^2 M_i^2$  in the RGE, the nucleosynthesis cannot be affected by the existence of a renormalization group induced CC in either of the  $\mu$ -frames (14) and (15), but if these terms are allowed the nucleosynthesis period could be jeopardized in the first frame (unless an additional fine tuning is arranged among the various  $T_n^2 M_i^2$  contributions [21]) but it would remain completely safe in the second frame. We have thus arrived from the nucleosynthesis analysis to a similar conclusion as before regarding the CC value at the contemporary epoch; viz. that the  $\mu$ -frame (14) is preferred to the (15) one for a consistent RG description of the CC evolution at these two crucial epochs.

## 6 On the running of the gravitational constant

Let us consider the running of the gravitational (Newton's) constant, which can be evaluated in the framework of the algorithm developed for the CC. From the quantum field theory point of view, the Hilbert-Einstein term should be introduced into the vacuum action (5), because otherwise the theory is not renormalizable. Then the renormalization condition for the gravitational constant could be implemented at the scale where it is measured experimentally, that is at the scale of the Cavendish experiment.

Along with the CC, the induced Hilbert-Einstein term is generated by exactly the same mechanism as the cosmological term. Disregarding the high derivative terms, we obtain from Eqs. (4) and (18) the action of induced gravity in the form (10) after replacing  $G_{vac} \rightarrow G_{ind}$  and  $\Lambda_{vac} \rightarrow \Lambda_{ind}$ , with  $G_{ind}$  defined by

$$\frac{1}{16\pi G_{ind}} = -\frac{\xi m^2}{f} \quad (39)$$

and  $\Lambda_{ind}$  given by (20). The physical observable value of the gravitational constant,  $G_{ph}$ , obtains from

$$G_{ph}^{-1} = G_{ind}^{-1}(\mu_c) + G_{vac}^{-1}(\mu_c). \quad (40)$$

When trying to analyze this equation the problem is that the value of the non-minimal parameter  $\xi$  is unknown. Indeed, since  $G_{ind}$  is (unlike  $G_{ph}$ ) unobservable, there is no a priori reasonable criterion to select a value for  $\xi$  at any given scale, while the scaling dependence of  $\xi$  is governed by a well-known renormalization group equation (see, for example, [12]). In the case of the SM we find

$$(4\pi)^2 \frac{d\xi}{dt} = \left[ \xi - \frac{1}{6} \right] \left( 6f - \frac{9}{2} g^2 - \frac{3}{2} g'^2 + 2 \sum_i N_i h_i^2 \right), \quad \xi(0) = \xi_0 \quad (41)$$

where the expression in the parenthesis is the very same one as in the equation for the mass in (23). We remark, that from the physical point of view there is no preference at which scale to introduce the initial data for  $\xi(t)$ , because this parameter cannot be measured in a direct way. Some formal arguments can be presented that  $\xi \approx \frac{1}{6}$  at high energies [30] and that it runs very slowly when the energy decreases [31]. As we shall see later on, the value of  $\xi$  is not very important for establishing the value of  $G_{ph}$  at different scales.

Now we are in a position to study the scaling dependence for the gravitational constant. As in the case of the CC, we must consider the vacuum and induced counterparts independently. The one-loop RGE for the vacuum gravitational constant can be computed e.g. with the help of the Schwinger-De Witt technique to extract the divergent part of the one-loop effective action, and has the form (see, e.g. [32])

$$(4\pi)^2 \frac{d}{dt} \frac{1}{16\pi G_{vac}} = 4m^2 \left( \xi - \frac{1}{6} \right) - \frac{1}{3} \sum_i N_i m_i^2, \quad G_{vac}(0) = G_0, \quad (42)$$

where the sum is taken over the spinor fields with the masses  $m_i$ , and  $t = \ln(\mu/\mu_c)$ . The value of  $G_0$  corresponds to the renormalization condition at  $\mu = \mu_c$  and must be chosen according to (40). The solution of (41) can be written in the form:

$$\xi(t) - \frac{1}{6} = \left( \xi_0 - \frac{1}{6} \right) U(t), \quad (43)$$

where  $\xi_0 = \xi(0)$  and  $U(t)$  is defined in (25). Then, the solution of (42) has the form

$$\frac{1}{16\pi G_{vac}(t)} = \frac{1}{16\pi G_0} + \frac{4}{(4\pi)^2} m_F^2 \left( \xi_0 - \frac{1}{6} \right) \int_0^t U^2(t) dt - \frac{1}{3(4\pi)^2} \sum_i N_i \int_0^t m_i^2(t) dt. \quad (44)$$

Thus, the scaling behavior of the parameter  $G_{vac}$  is determined, with accuracy to the integration constant  $G_0$ , by the scaling behavior of the couplings and masses of the matter fields, and by the initial unknown value  $\xi_0$ .

Consider the induced part. The effective potential of the Higgs field, with the non-minimal term (4), is given by a loop expansion:

$$V_{eff} = V_{cl} + \sum_{n=1}^{\infty} \hbar^n V^{(n)}, \quad (45)$$

where the classical (tree-level) expression

$$V_{cl} = -\frac{1}{2}(m^2 + \xi R) \phi^2 + \frac{f}{8} \phi^4 \quad (46)$$

is seen to get an additional contribution from curvature. Since we are interested only in the running, it is not necessary to account for the renormalization conditions and one can simply take the renormalization group improved classical potential. It can be easily obtained from (46) if all the quantities  $\phi, f, m^2, \xi$  are replaced by the corresponding effective charges. The gauge ambiguity related to the anomalous dimension of the scalar field and consequently with the running of  $\phi$  is fixed by the relation (18). Taking into account (39), we arrive at

$$-\frac{1}{16\pi G_{ind}(t)} = \left[ \frac{1}{6} + \left( \xi_0 - \frac{1}{6} \right) U(t) \right] m_F^2 U(t) f^{-1}(t), \quad (47)$$

where  $f(t)$  is solution of Eq. (23). The formulas above give the scaling dependence for the induced gravitational constant, which is completely determined by the running of  $m^2(t)$ ,  $f(t)$  and  $\xi(t)$ . Eq. (40) enables one to establish the scale dependence of the physical gravitational constant:

$$-\frac{1}{16\pi G_{ph}(t)} = -\frac{1}{16\pi G_0} + \left[ \frac{1}{6} + \left( \xi_0 - \frac{1}{6} \right) U(t) \right] \frac{m_F^2}{f(t)} U(t) - \frac{4m_F^2}{(4\pi)^2} \left( \xi_0 - \frac{1}{6} \right) \int_0^t U^2(t) dt + \frac{1}{3(4\pi)^2} \sum_i N_i \int_0^t m_i^2(t) dt. \quad (48)$$

From the last formula follows, that the scaling dependence of the inverse gravitational constant (deviation of its value from  $G_0$ ) is proportional to  $M_F^2 \sim 10^5 \text{ GeV}^2$  whereas the observable value of  $G_{ph}^{-1}$  is  $M_{Pl}^2 \sim 10^{38} \text{ GeV}^2$ . Hence, the only one chance to have relevant running of  $G_{ph}$  is to consider the theory with huge  $\xi$  comparable with  $10^{33}$ . It is easy to see that this can be inconsistent with our general supposition in Eq.(46) that  $\xi R$  is small as compared to  $m^2$  in the Fermi epoch. We use this fact when take the flat-space formula (17) for the SSB, despite our potential contains a non-minimal term (4). In order to justify this, one has to remind that the values of Ricci tensor and energy-momentum tensor are linked by Einstein equations (11). The typical value of the components of the physical  $T_{\mu\nu}$  is  $\mu^4$  where  $\mu$  is the scale at the corresponding epoch. At the Fermi epoch,  $\mu \sim m \sim 100 \text{ GeV}$ , the typical value of the components of  $R_{\mu\nu}$  is of order  $R \sim 8\pi G T_\mu^\mu \sim 8\pi m^4/M_P^2$ , and so indeed our approximation (17) works well only if

$$|\xi| \ll \frac{1}{8\pi} \left( \frac{M_P}{m} \right)^2 \sim 10^{33}. \quad (49)$$

On the other hand, since the scale dependence of  $\xi$  (43) is not strong [31],  $\xi$  must be close to  $1/6$  also at the present epoch. If being of the order  $10^{33}$ ,  $\xi$  should manifest itself in the low energy phenomena, and since this is not the case we will not consider this possibility. For values of  $\xi$  satisfying (49) the running of  $G_{ph}$  is negligible so that the exact value of  $\xi$  is not important for the definition of the gravitational constant.

Finally, we remark that the negligible running of  $G_{ph}$  that we have found in our framework, is very much welcome in order not to disturb primordial nucleosynthesis, which in fact can only tolerate few percent deviations of  $G_{ph}$  with respect today's value [20]. This feature, together with the already proven relative smallness of the vacuum energy as compared to the energy density of matter at that epoch, is quite rewarding for the physical consistency of our approach.

As for the potential heavy mass corrections to the previous analysis, similar considerations can be made here in complete analogy with the CC case. However, in the present case the dimensionality of  $G_{ph}^{-1}$  enforces the additional terms to be of the form  $(\mu/M)^{2n} M^2$  ( $n = 1, 2, \dots$ ). Hence the first, and most important, correction is just  $\mu^2$  (independent of the heavy mass  $M$ ). Then, it is obvious that the presence of these terms would be innocuous because  $\mu^2 \ll G_{ph}^{-1}$  for both  $\mu$ -frames (14) and (15).

## 7 Cosmological constant and inflation

It is well known that many problems of standard cosmology can be solved via the inflationary paradigm [19]. Then, it is common wisdom to suppose that some kind of induced CC was responsible

for the inflation. But, in the phenomenologically acceptable scenarios, one can achieve the successful inflation through the inflaton models [19], while the origin of the inflaton remains unclear. There is an alternative scenario of inflation, which originates from the quantum vacuum effects of matter fields [33, 34, 30]. These effects are related to the renormalization of the higher derivative sector in (5) and do not suppose the crucial role for the cosmological constant. However, it is interesting to check, how the CC affects the inflationary solution.

In order to arrive at the anomaly-induced effective action, one has to start from the quantum theory of matter fields which are conformally coupled to gravity. In the framework of the cosmon model [28] one can admit the existence of the masses, but in any case one has to request the  $\xi = 1/6$  condition [35]. Then the high derivative sector of the vacuum action (5) can be reduced to

$$S_{vac} = \int d^4x \sqrt{-g} \left( l_1 C^2 + l_2 E + l_3 \square R \right),$$

where,  $l_{1,2,3}$  are some parameters,  $C^2$  is the square of the Weyl tensor and  $E$  is the integrand of the Gauss-Bonnet topological invariant. the renormalization of the action (7) leads to anomaly

$$T = \langle T_\mu^\mu \rangle = - (w C^2 + b E + c \square R), \quad (50)$$

where  $w, b, c$  are the  $\beta$ -functions for the parameters  $l_1, l_2, l_3$  <sup>11</sup>

$$w = \frac{N_0 + 6N_{1/2} + 12N_1}{120 \cdot (4\pi)^2}, \quad b = -\frac{N_0 + 11N_{1/2} + 62N_1}{360 \cdot (4\pi)^2}, \quad c = \frac{N_0 + 6N_{1/2} - 18N_1}{180 \cdot (4\pi)^2}. \quad (51)$$

The anomaly-induced action, for the massless fields has the following form [36]:

$$\bar{\Gamma} = S_c[\bar{g}_{\mu\nu}] + \int d^4x \sqrt{-\bar{g}} \{ w \bar{C}^2 + b(\bar{E} - \frac{2}{3} \bar{\nabla}^2 \bar{R}) + 2b \bar{\Delta} \sigma + d \bar{F}^2, \quad (52)$$

where we have included the contribution of the vector fields. Here the original metric is decomposed according to  $\bar{g}_{\mu\nu} = g_{\mu\nu} e^{2\sigma}$  and  $S_c[\bar{g}_{\mu\nu}]$  is the conformal invariant part of the quantum contribution to the effective action, which is an integration constant for the solution (52). Adding up (52) with the Hilbert-Einstein term (10) and performing the variation of the total action with respect to  $a(t) = e^{\sigma(t)}$  ( $t$  is the physical time) one obtains the corresponding equation of motion [30]:

$$E[a] = a^2 \ddot{a} + 3a \dot{a} \ddot{a} + \left( 5 - \frac{4b}{c} \right) \dot{a}^2 \ddot{a} + a \dot{a}^2 - \frac{2M_P^2}{c} a \left( a \ddot{a} + \dot{a}^2 \right) = 0 \quad (53)$$

This equation leads to the exponential solution of Starobinsky [33]

$$a(t) = e^{H_P t}, \quad H_P = \frac{M_P}{\sqrt{-b}}, \quad (54)$$

which is stable for the particle content  $N_1$  vectors,  $N_{1/2}$  spinors and  $N_0$  scalars satisfying the inequality

$$N_1 < \frac{1}{3} N_{1/2} + \frac{1}{18} N_0. \quad (55)$$

The relation (55) allows one to distinguish between very different scenarios of non-stable inflation [33, 34] and the stable one [30]. Our purpose here is to verify how the solution (54) and the stability

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<sup>11</sup>The contribution of the Weyl spinor is half of that for the Dirac spinor and the contribution of complex scalar is twice the one of the real scalar.

condition (55) look in the presence of the cosmological constant. Let us formulate a more general question: how will Eq. (53) modify in the presence of some source term (matter or CC) which satisfies some equation of state? It is easy to see that any matter content produces an energy density which rapidly decrease with the growth of  $a(t)$ . Substituting such a source term into (53), and making numerical analysis, we find that the presence of such source, even with initial energy density of the Planck order of magnitude, does not perturb in any essential way the stable solution (54)<sup>12</sup>. This fact indicates, that the "equation of state" for the high derivative terms in (52) is such that the corresponding energy density does not change with  $a(t)$ . Since the same property is shared by the CC, one can expect that the CC will be the only possible source of modifications for the stable inflationary solution. Let us check this. The equation with the CC source has the form<sup>13</sup>:

$$E[a] = -\frac{2\Lambda_{ph}}{3c} a^3, \quad (56)$$

where the operator  $E[a]$  has been defined in (53). Taking into account that the logarithmic dependence is weak, for the sake of simplicity we consider that  $\Lambda_{ph}$  is constant with respect to  $a$ . Substituting the exponential function  $a(t) = e^{Ht}$  into (56), we arrive at two solutions for the induced gravity with the CC (see also [37] where similar solutions have been found):

$$H_1^2 = \frac{1}{2} H_P^2 + \frac{1}{2} \sqrt{H_P^2 + \frac{2}{3b} \Lambda_{ph}}, \quad H_2^2 = \frac{1}{2} H_P^2 - \frac{1}{2} \sqrt{H_P^2 + \frac{2}{3b} \Lambda_{ph}}. \quad (57)$$

In the case of the sufficiently small CC the first solution is close to the inflationary Starobinsky solution (54). At the inflationary epoch, the CC could include the contributions from the running due to the massive constituents of GUT, and also due to the phase transitions at various scales. But, for the consistency of the semiclassical approach [30] one has to suppose that all the particle masses are much smaller than  $M_P$ . Therefore the first solution in (53) manifests just a small difference with  $H_P$ , such that we always have  $H_P^4 \propto M_P^4$ . The sign of  $\Lambda_{ph}$  does not play much role in this context.

The second solution is also interesting, because it substitutes the flat static solution  $a(t) \equiv a_0 = \text{const}$  of Eq. (53). This solution applies at the very low energy. For example, at the cosmic scale the value of  $\Lambda_{ph}$  is positive and of the order of  $\rho_M^0$ . Let us consider this solution in details, disregarding the effect of matter density (which will be considered elsewhere). In the presence of the CC this flat solution becomes some very slow inflation, such that the effect of the anomaly-induced term is not seen. In order to check the last statement, we remind that the CC is very small  $\Lambda_{ph} \ll M_P^4$ . Expanding the second solution  $H_2$  up to the first order we find that the effective exponential behavior is governed by  $H_{eff} = \Lambda_{ph}/(6M_P^4)$ . It is easy to check that this is exactly the value which emerges in the Einstein theory with the CC but without higher derivative induced terms (52). Of course, the inflationary solution exists only for the positive  $\Lambda_{ph}$ .

Let us discuss the stability of the two solutions (57) under the perturbations of the conformal factor of the metric  $a(t)$ <sup>14</sup>. For this purpose, one has to expand, in both cases,

$$H(t) = H_{1/2} + x(t) \quad (58)$$

<sup>12</sup>Authors are grateful to Ana Pelinson for discussion of this point.

<sup>13</sup>In this equation we disregard the scale dependence of the CC. This is justified, because this dependence has logarithmic structure and the result can not change too much if we use another approximation for the  $\Lambda_{ph}(a)$ .

<sup>14</sup>The detailed investigation of the stability problems will be given elsewhere [38].

and, after linearization, investigate the asymptotic behavior of the perturbation  $x(t)$ . It is easy to see, that for the physically interesting case  $\Lambda_{ph} \ll M_P^4$  the stability condition for the first solution  $a_1(t) = e^{H_1 t}$  is nothing but (55). This is quite natural, because the cosmological constant does not play much role in this solution. With the opposite sign of the inequality in (55), the second solution is stable, exactly as its flat counterpart in the zero CC case.

The last observation concerns the possibility to have many copies of a very light scalars which we have discussed by the end of section 4. Since the presence of the CC does not change the stability condition (55), the existence of too many scalars would unavoidably produce a very fast inflation. Since today only the photon (among the known particles) remains active, the condition (55) predicts that the present-day Universe must live close to the non-inflationary stable state. But, if we suppose that there are more than 18 super-light scalars, they would not decouple below  $\mu_c$ , the Universe would be inflating at the corresponding scale and this could break the nucleosynthesis. Thus, these scalars are ruled out by observations.

Finally, as in the  $G_{ph}^{-1}$  case, we point out that the potential heavy mass contributions in the RGE are also of residual nature here because  $\mu^2 \ll H_P^2$  for both  $\mu$ -frames (14) and (15), so that they cannot alter in any significant way the conclusions derived from eqs. (57).

## 8 Conclusions

We have considered several new aspects of the CC problem using the RG method. The main difference between the CC and the other SM parameters is that the input data on the CC are only known at the cosmic scale  $\mu_c$  defined in (8), and therefore the CC can only be renormalized at this scale, whereas the other parameters are renormalized at the Fermi scale  $M_F \gg \mu_c$ . This hints at the necessity for the unnaturally great precision of the vacuum CC value at  $\mu = \mu_c$ . If we take the cancellation of the vacuum and induced CC's at  $\mu = \mu_c$  as an experimental fact, the standard quantum field theory framework predicts the running of the CC above the scale  $\mu_c$ , and also restricts the kind of new (massive) particles potentially existing below that scale.

Our investigation shows that the running of the CC could take place as the expanding universe progressively cooled from energies comparable to the Fermi scale (or above) down to the modern epoch, but in actual fact the running itself got stuck at the scale  $\mu_c$ , which is comparable to the lightest neutrino mass,  $m_{\nu_1} \sim 0.001 \text{ eV}$ . This scale is many orders of magnitude greater than the present-day cosmic scale, defined by the value of Hubble's constant  $H_0 \sim 10^{-33} \text{ eV}$ . Still, the range of the running of the CC is of the same order of magnitude ( $\Lambda_{ph} \sim 10^{-47} \text{ GeV}^4$ ) as the energy density of matter in the present Universe. The existence of ultralight (sub-gravitationally coupled) degrees of freedom below the scale  $H_0$  is not, in principle, excluded. However, they cannot be responsible for a vanishing CC in the far IR through the RG mechanism. Finally, we may get a better understanding of the aforementioned "cosmic coincidences" as follows. On the one hand  $\rho_M^0 \sim \rho_c^0$ , and on the other it turns out that  $m_{\nu_1}^4 \sim \rho_c^0$ . In our framework these two facts can be reconciled with the third coincidence,  $\rho_M^0 \sim \Lambda_{ph}$ , because the CC must be generated through the RG running, and naturally acquires a value comparable to the fourth power of the mass of the lightest (massive) degree of freedom:  $\Lambda_{ph} \sim m_{\nu_1}^4$ . Whether the neutrino is ultimately responsible for this value cannot be established at the moment, but the correct (positive) sign of the CC may require the existence of a light scalar of comparable mass. Recall that values of the CC significantly greater than  $m_{\nu_1}^4$  might contradict well-known anthropic considerations [1, 10].

Another important conclusion of our semiclassical RG analysis of the CC problem is the fol-

lowing: Of the two possible choices (14) and (15) for the scale  $\mu$ , definition (14) seems to be the most appropriate one for a consistent description. First, (14) sets the natural scale of the graviton energy as read off Einstein equations applied to the FLRW metric. Second, if heavy mass corrections to the sharp cut-off RG analysis need eventually be included, definition (14) is the only one that allows to introduce the appropriate modifications in the RGE in a way that is naturally compatible (i.e. without any additional fine tuning) with both the value of the CC at present and the value of the CC at the epoch of the nucleosynthesis. Moreover, the existence and adequacy of the  $\mu$ -frame (14) shows that for the semiclassical RG analysis of the cosmological parameters we must actually distinguish between two energy scales: One scale  $\mu$  refers to the typical energy of the external gravitons, and another scale  $\mu'$  refers to the typical energy of the external matter particles – the latter being closer to the frame (15). While the former is relevant for the running of the cosmological parameters the latter characterizes the scaling evolution of the SM couplings and masses. Since the two scales  $\mu$  and  $\mu'$  typically run on vastly different regimes ( $\mu \sim H$  and  $\mu' \sim M_F$ ), we are unavoidably confronted with the necessity of an apparent large fine tuning when we add up the induced and vacuum contributions to the CC in order to build up the physical value defined in eq.(16):  $\Lambda_{ph} = \Lambda_{vac}(\mu = \mu_c) + \Lambda_{ind}(\mu' = \mu_c)$ . The fine tuning is prompted by the fact that the CC renormalization point  $\mu_c$  lies in the vicinity where  $\mu$  typically runs while it is exceedingly away from the typical range of  $\mu'$ . The two scales can never be close because the graviton energy is always suppressed by a factor  $M_P$ , more precisely:  $\mu/\mu' \sim \mu'/M_P \sim M_F/M_P$ .

Similarly to the CC, there is a running of the gravitational constant. We have proven that the scope of this effect is small and that it can not spoil primordial nucleosynthesis. Furthermore, we have found that the CC does not distort the virtues of the anomaly-induced inflation. At high energies the presence of the CC does not modify, in any essential way, neither the velocity of inflation nor the condition of stability (55). Finally, at low energies the anomaly-induced terms do not change the observed value of the CC.

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